

Relationship between the static and dynamic elastic modulus of brick masonry constituents

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HIGHLIGHTS

- Reliable estimates of dynamic elastic modulus can be obtained quickly and easily.
- Static elastic modulus is preferred for most common structural verifications.
- Empirical expression proposed to estimate the static modulus from the dynamic one.
- Expression tested on a diverse set of typical brick masonry constituents.

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ABSTRACT

Different brick masonry typologies have been and are still widely used in construction worldwide. Despite being a fundamental deformation property, experimentally determining the static elastic modulus in compression for brick masonry constituents remains a challenging task. Static modulus estimates usually show much larger dispersion than those involved in determining the dynamic elastic modulus. Although the static property is preferred for common structural verifications, the relationship between the two is yet to be well understood. In light of the above, this paper provides an empirical expression to estimate the static elastic modulus of brick masonry constituents from its dynamic counterpart.

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1. Introduction

The elastic modulus of a material is a fundamental property that is crucial for characterising the deformation behaviour of structural elements whether it is with respect to the design of new structures or for the assessment of existing ones. Since typical brick masonry constituents usually have negligible or very low tensile strengths, the elastic modulus in compression is often the most relevant material property related to elastic deformation.

Experimental techniques that can be used to evaluate this property may be classified as static or dynamic. The former involves directly loading a specimen and measuring the corresponding change in strain. The static elastic modulus (E_{st}) is then computed by evaluating the slope of the experimental stress–strain curve in

the elastic deformation range. On the other hand, the dynamic elastic modulus (E_{dy}) can be derived from the measured resonant frequency of a specimen in a specific vibration test or from the measured velocity of a stress wave passing through the material. It is now well known that E_{st} can differ significantly from E_{dy} , with the latter generally being greater. It can be envisaged that this empirically known inequality arises mainly due to the fact that E_{dy} is measured at almost negligible stress levels compared to its static counterpart [1]. However, studies have shown that this discrepancy is also due to the inherent heterogeneity of materials causing them to respond differently under cyclic or vibratory loading conditions [1–5].

For brittle materials, such as most typical brick masonry constituents, the static methods present significant challenges. Firstly, they are often very time consuming since they require gradually loading carefully prepared cylindrical or prismatic samples [6]. Secondly, deformation magnitudes in the elastic range tend to be very low and can change relatively abruptly during loading due

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to the nature of the material [7–10]. Moreover, brick masonry constituents can contain noticeable heterogeneities that can significantly skew estimates of the deformation [11]. Finally, most transducers that can be used to measure surface strains of such materials tend to be very sensitive to machine-specimen surface interaction [12]. As such, it can be very difficult to obtain reliable measurements that reflect the actual elastic deformation of the material and the scatter of results is usually high, as evidenced in [7–10,13].

Contrarily, the dynamic methods are much faster to execute and have the added benefit of being non-destructive. In addition, they do not suffer from the same limitation related to the difficulty of accurately capturing representative deformations. As a result, most of the scatter in experimental results from such tests can usually be attributed to heterogeneity of the sample set rather than experimental error [11]. However, for most common structural calculations, the statically determined modulus is preferred over that obtained by dynamic methods since the former is more representative of actual loading conditions. Given this fact, it is understandable why the correlation of these two parameters for brittle materials has received considerable attention, most notably for Portland cement concrete and for rocks.

Due to its widespread use as a construction material during the 20th century, a substantial research effort has been dedicated to better understanding the relationship between E_{dy} and E_{st} for concrete. In fact, in 1972, the empirical relationship shown below was even included in the now superseded British code of practice for the structural use of concrete [14].

$$E_{st} = 1.25E_{dy} - 19 \quad (1)$$

Where E_{st} and E_{dy} refer respectively to the static and the dynamic elastic modulus expressed in GPa.

It should be noted that this expression is not applicable for concretes with a cement content greater than 500 kg/m³ or for light-weight concrete [1]. To address this limitation, some researchers proposed the following expression for the latter [15].

$$E_{st} = 1.04E_{dy} - 4.1 \quad (2)$$

A simpler general empirical relationship for concrete has also been proposed [16]:

$$E_{st} = 0.83E_{dy} \quad (3)$$

As previously mentioned, the inherent material heterogeneity of concrete affects the two moduli (E_{st} and E_{dy}) in different ways. As such, studies have also been carried out in order to better understand how different material properties, such as compressive strength (f_c) or density (ρ_c), can influence the relationship between E_{st} and E_{dy} . As a result of this effort, it has been found that for concrete, the ratio of E_{st} to E_{dy} usually increases with increasing f_c [17,1,18]. Many researchers have also attempted to develop empirical relationships between E_{st} and E_{dy} that also incorporate other physical parameters. Although many of those ended up having a relatively limited range of applicability, one of the most useful expressions proposed for concrete does indeed suggest that the relation between E_{st} and E_{dy} is a function of density [19]:

$$E_{st} = \frac{446.09 \cdot E_{dy}^{1.4}}{\rho_c} \quad (4)$$

Where E_{st} and E_{dy} are once again to be specified in GPa and ρ_c is the density of hardened concrete in kg/m³.

Because of its relevance in the field of geomechanics, the relation between the static and dynamic elastic modulus of rocks has also received considerable attention [20,21,6,22–30]. However, the empirical relationships proposed by most authors are either applicable to only certain types of rocks or are valid for a limited

elastic modulus range that tends to be greater than the elastic moduli of most typical brick masonry constituents. Of the several empirical relations available for rocks in the scientific literature, the following one proposed by Eissa and Kazi [6] could possibly lend itself to the case of brick masonry constituents since it was derived from a sample containing one of the most diverse set of rocks. Similarly to the expression proposed by Popovics for concrete (Eq. (4)), the bulk density is also included as an explanatory variable in this relation.

$$\log_{10}E_{st} = 0.77\log_{10}(\rho_r E_{dy}) + 0.02 \quad (5)$$

E_{st} and E_{dy} refer to the static and dynamic moduli of the rock in GPa while ρ_r refers to its bulk density in g/cm³.

Although the sample set used for the derivation of Eq. (5) consists of a very diverse set of rocks, the compiled data used for the analysis also come from a wide variety of sources. It could therefore not be ensured that testing conditions have been kept constant for all specimens included in the analysis. It is well known that testing conditions can have a significant effect on the final estimated static or dynamic elastic modulus. A more recent study [30], based on a dedicated experimental campaign on 33 specimens coming from 8 different igneous, sedimentary and metamorphic rock types, proposes the following relationship instead.

$$E_{st} = 11.531\rho_r^{-0.457}E_{dy}^{1.251} \quad (6)$$

With the static and elastic moduli expressed in GPa and the bulk density expressed in kg/m³.

The same authors also propose additional correlation models that incorporate total porosity and compressive strength of the rocks as additional explanatory variables. They report improved goodness of fit metrics with increasing level of complexity [30].

In spite of the many empirical relationships proposed for concrete and rock, and despite the widespread use of brick masonry in construction, there exists very little research that attempts to explore this relationship for the case of brick masonry constituents. Totoev and Nichols do compare the static and dynamic moduli for some brick types [31,32], but no relationship is proposed for practical applications. As such, the relationship is still not well understood for the case of brick masonry constituents.

The main aim of this research is thus to propose an expression that can be used to estimate the static elastic modulus of typical brick masonry constituents from the dynamic one for practical applications. The experimental study includes different types of bricks and mortars so that useful ranges of results for different brick masonry typologies could be derived. First, the procedures used for the experimental determination of the elastic moduli of all the specimens are presented. The static elastic moduli are then compared to the dynamic ones evaluated for the same specimens. Finally, the suitability of several of the empirical expressions developed for rocks and concrete are assessed before proposing one to be used for typical brick masonry constituents.

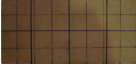
2. Experimental program

The experimental campaign was carried out at the Laboratory of Technology of Structures and Building Materials of the Universitat Politècnica de Catalunya (UPC-BarcelonaTech). This section presents information about the material components and the procedures employed for the experimental determination of the static and dynamic elastic moduli.

2.1. Materials tested

Seven different groups of solid bricks and two different types of mortar were tested as part of the experimental campaign. Five of

Table 1
Groups of brick types tested.

Group	Number of bricks	Number of specimens	Manufacturing	Year	f_{cn}^* [MPa]	Average bulk density [kg/m ³]	Sample view
I(a)	7	42	Handmade in moulds	2017	16.1 ± 16%	1,781 ± 1%	
I(b)	6	36	Handmade in moulds	2015	17.0 ± 15%	1,768 ± 3%	
II	5	15	Conventional extrusion	2016	40.0 ± 16%	1,655 ± 0.3%	
III	3	18	Handmade in moulds	1903	8.0 ± 17%	1,598 ± 5%	
IV	6	18	Conventional extrusion	2018	53.2 ± 8%	1,673 ± 0.4%	
V(a)	2	11	Handmade in moulds	1930	8.3 ± 43%	1,720 ± 1%	
V(b)	6	16	Handmade in moulds	1930	10.7 ± 15%	1,718 ± 1%	

* Reference normalised compressive strength for corresponding brick type obtained by testing bricks as prescribed in the European standard EN 772-1 [33].

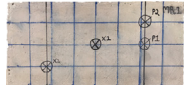
the seven brick groups were made up of handmade bricks formed by moulding (I(a), I(b), III, V(a), and V(b)). Groups I(a) and I(b) consisted of new solid terracotta bricks while bricks from groups III, V(a) and V(b) were extracted from existing structures in Barcelona. Specifically, group III bricks were acquired from an early 20th Century industrial complex while bricks from groups V(a) and V(b) were extracted from a typical residential building [11]. The remaining 2 brick groups (II and IV) consisted of new solid bricks manufactured using a conventional extrusion process. All brick specimens used for the static tests consisted of 40 × 40 × 80 mm³ prisms cut out from whole bricks (see Section 2.2.1). A brief summary of the different groups of bricks tested is given in Table 1.

The mechanical properties of the two types of mortar considered differed significantly. The first mortar type consisted of a hydraulic lime weakened by adding recycled limestone filler to the mixture in order to match mechanical properties more representative of mortars found in historical constructions (MB and MIIB) [34]. For both mortars MB and MIIB, 50% of the powder volume was replaced by the limestone filler. The main difference between specimens from these two groups is that those from group MB were unmoled 5 days after initial casting whereas those from group MIIB were unmoled after 14 days. This change was implemented because specimens from group MB were found

to be too fragile at the time of unmoleding [11]. Specimens from group MB were tested 32 days after initial casting while MIIB specimens were tested after 27 days. The second mortar type considered consisted of a typical cement mortar used in new constructions (MC). This type of mortar was chosen because it can serve as a good control sample, not only due to the many studies that have been carried out on the properties of Portland cement mixes, but also because it is relatively easy to prepare homogeneous and isotropic specimens from this material. Since several studies [35,36] reveal that such mixes have usually already gained between 85% and 90% of their stiffness after just 4 days, it was deemed suitable to test MC specimens after 14 days. All mortar specimens for the static tests have been prepared in standard 40 × 40 × 160 mm³ moulds. Key characteristics of the different mortar specimen groups tested are summarised in Table 2.

Some additional information on the brick and mortar specimen groups are presented in [11] which describes the procedure used to obtain the dynamic elastic moduli compared to the static ones in this study. With respect to mortar specimens, the procedure described in [11] consisted of non-destructive tests carried out on specifically moulded brick-shaped specimens, while the static tests were carried out on specimens cast at the same time in separate standard prismatic moulds. As for the bricks, the procedure described in [11] consisted of non-destructive tests executed on

Table 2
Groups of mortar types tested.

Group	Number tested	Mix proportions (by weight)	f_{cn}^* (28 days) [MPa]	Average bulk density [kg/m ³]	Sample view
MB	3	Lime: filler: water 1: 0.64: 0.37	2.3 ± 6%	1,776 ± 2%	
MIIB	3	Lime: filler: water 1: 0.64: 0.37	2.0 ± 12%	1,932 ± 1%	
MC	6	Cement: sand: water 1: 3.2: 0.33	48.8 ± 5%	2,183 ± 1%	

* Reference normalised compressive strength for corresponding mortar type obtained by testing prismatic specimens (cured for 28 days) in accordance with the European standard EN 1015-11 [37].

whole brick specimens, whilst the static tests were carried out on small specimens cut out from the same bricks. It should be noted that the average compressive strength (f_{cn}) listed in [11] for some brick specimen groups differs from the one shown in Table 1. This difference mainly arises from the fact that the f_{cn} reported in [11] was derived from other different bricks of the same type but not from the actual specimens tested during the experimental campaign. However, as described in Section 2.2.1, the f_{cn} reported in Table 1 were derived from specimens cut out from the same bricks as the ones used to determine the static and dynamic elastic modulus.

2.2. Experimental determination of static modulus

2.2.1. Specimen preparation for static tests

Static tests from which the elastic moduli are to be evaluated involve fixing the applied stress at lower levels than the compressive strength for fixed intervals during testing. Hence, prior knowledge of the approximate compressive strengths of the specimens from each group is required before executing the tests.

Before cutting out any prisms from bricks to perform static tests, the two opposing largest faces of the bricks were polished so that each brick had a uniform thickness of 40 mm. Since handmade bricks formed by moulding can be reasonably approximated as being isotropic [11,38], it can be assumed that for each specimen group, the average compressive strength evaluated across the brick lengths should correspond to that evaluated across the widths. As such, for specimen groups I(a), I(b), III and V(a), a $40 \times 40 \times 80 \text{ mm}^3$ prism across the width was cut out from each brick to estimate the compressive strength according to the procedure in [33] (see (c) in Fig. 1). The reference compressive strength (f_{cn}) for each group was then taken as the average of the ones estimated from these prisms. As shown in Fig. 1, three more $40 \times 40 \times 80 \text{ mm}^3$ prisms were cut out across the width of each remaining brick to evaluate the static elastic modulus across the width ($E_{st,W}$), while three equally sized prisms were cut across the length to evaluate the modulus in that direction ($E_{st,L}$).

It should be noted that one of the prisms across the width of a brick from group V(a) was damaged and therefore only 2 specimens could be used to evaluate $E_{st,W}$ for that particular brick. Furthermore, although bricks from group V(b) were also formed by moulding, the same specimens shown in Fig. 1 were not cut out from each brick. This is due to the fact that $40 \times 100 \times 100 \text{ mm}^3$ prisms had already been cut from these bricks to evaluate their compressive strengths according to [33] as part of a pre-

vious experimental campaign. Naturally, the compressive strength evaluated from the previous campaign was taken as the representative strength for each brick from this group. However, this meant that less remaining area was available from each brick to extract specimens for the evaluation of elastic moduli. As such, only $40 \times 40 \times 80 \text{ mm}^3$ prisms across the length were cut from each brick. Although the number of specimens that could be cut out from each brick varied, it was ensured that at least 2 were extracted from each to calculate $E_{st,L}$.

As described in greater detail in Section 2.3, only the dynamic elastic modulus across the length is available for bricks produced by extrusion (groups II and IV). Hence, only $E_{st,L}$ is included in this study for these bricks. These were also derived from tests carried out on $40 \times 40 \times 80 \text{ mm}^3$ specimens cut out across the length of each brick.

Given that bricks are most often loaded in compression across their thickness, the choice of investigating the relationship between static and dynamic modulus across the width and length of bricks might appear counter-intuitive. Particularly when the procedure described in [11] allows for the estimation of dynamic elastic moduli of bricks handmade in moulds across their thickness. However, since this study mainly aims to better understand the relationship between static and dynamic moduli, this choice was made to ensure that only reliable estimates that reflect the actual material behaviour are compared. As stated in Section 1, for typical brittle brick materials, measuring surface strains in the elastic range can be a very challenging task with several possible sources of error. The impact of these errors on the final estimated value of the static elastic modulus is likely to be significantly greater if the strains are measured over a shorter gauge length. Since the polished bricks tested as part of this research were at most 40 mm thick, only very short gauge lengths were available to measure strains across this dimension. In fact, the coefficients of variation of static elastic moduli estimated from strain measurements across the lengths and widths of bricks already proved to be significantly greater than those estimated from dynamic methods (see Section 3), despite the fact that they allowed strain measurements over a longer gauge length. Moreover, the dynamic modulus across the thickness is not available for the anisotropic bricks produced by extrusion. As such, elastic moduli estimated across the thickness of bricks were not included in this study.

Before testing, the surfaces of all the cut prismatic brick specimens were polished and regularised to ensure uniform loading and to reduce possible sources of error in deformation measurements. Since moisture content of the specimens can alter the

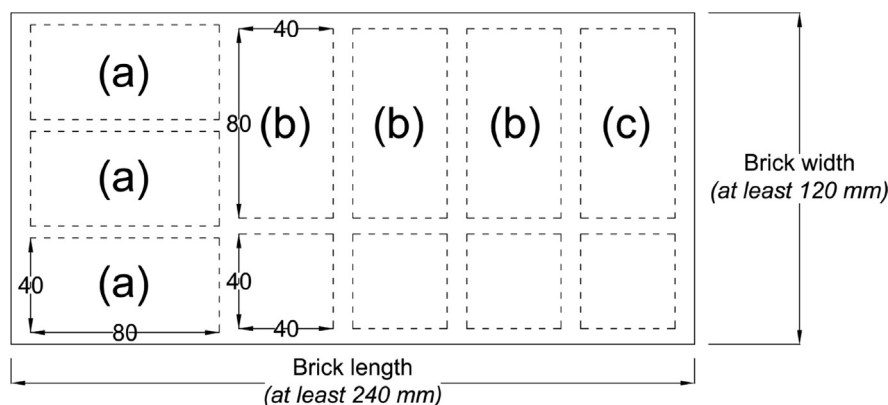


Fig. 1. Specimens cut from each handmade brick. (a) Specimens used for estimating static elastic modulus across length. (b) Specimens used for estimating static elastic modulus across width. (c) Specimen used for estimating compressive strength.

deformation behaviour of these materials, all specimens were dried at 120 °C in a drying oven until the mass was constant prior to testing.

In the case of the mortar specimen groups, standard 40 × 40 × 160 mm³ prismatic specimens were prepared and tested to evaluate both the compressive strength and the static elastic modulus. These were cast and tested at the same time as the brick-shaped specimens used for dynamic testing in [11].

2.2.2. Experimental procedure for static tests

The stress–strain relationship of brittle materials, such as typical brick masonry constituents, usually reveals a certain degree of non-linearity even at stress levels well below their ultimate strength. This can be attributed to microcracks [39] and creep effects [1]. Due to this inherent non-linearity, different definitions of the static elastic modulus have been proposed for such materials depending on how the latter is calculated. It is possible to find a *tangent modulus* at any point on the stress–strain curve, but this will only apply to loading levels in the vicinity of the point at which the modulus was calculated and is therefore of little practical significance. The *chord modulus* taken as the slope of the line connecting two specific points on the stress–strain curve is usually preferred since it is more representative of the material behaviour under actual loading conditions of interest. It should be noted that since the first point used for the computation of the chord modulus is usually at a very low stress level, some books [1], articles [7,9] and even standards [40] have come to refer to it as a *secant modulus*.

Naturally, since the secant modulus will depend on the two points chosen to compute it, it is important to select them carefully. To the best of the authors' knowledge, no standards exist for the determination of the static elastic modulus of brick masonry constituents. However, some guidance for the selection of appropriate nominal stress levels can be found from the European Standards for the determination of the static moduli of elasticity of concrete (EN 12390-13:2013) [40] and natural stone (EN 14580:2005) [41], as well as in the standard for the determination of the compressive strength of masonry (EN 1052-1:1998) [42]. All these standards recommend to use a nominal upper stress level at a third of the estimated compressive strength (f_c). However, they differ in their recommendation for the nominal lower stress level at which the strain should be evaluated for the computation of the static elastic modulus. Whilst the standard for concrete recommends a nominal lower level between 10% and 15% of f_c , that for natural stone recommends one corresponding to approximately 2% of f_c . Based on these recommendations, for the static tests carried out as part of this research, the nominal upper stress level (σ_b) was set as approximately equal to 30% of the representative compressive strength for each specimen group. The nominal lower stress level (σ_a) was set as approximately equal to 10% of the representative compressive strength for each group. The latter value was chosen because the measured strains at lower stress values approached the minimum resolution of the strain transducer for some of the specimens with low compressive strengths.

To eliminate some of the creep effects and to allow the extension pieces of the transducers to settle at fixed points on the material, it is vital to subject each specimen to cycles of pre-loading up to the nominal upper stress level. Standards [40,41] suggest to carry out three loading cycles and to determine the stabilised secant modulus based on the third cycle. This procedure was thus adopted for all the tests carried out to determine the static elastic modulus of the brick masonry constituents, resulting in the loading scheme shown in Fig. 2. As shown, the load was held constant for 60 s at the nominal upper and lower stress levels. The final increas-

ing branch of the loading scheme after the third cycle was implemented to verify the continuity of the slope of the stress–strain curve beyond σ_b .

It should be noted that specimens from group I(a) were tested before the final testing protocol was developed. For tests on these specimens, σ_a was set at 5% of f_c instead of 10%.

The experimental procedure thus involved applying a compressive load according to the loading scheme shown in Fig. 2 while recording vertical strain measurements across the surface of three faces using a gauge length of 50 mm as shown in Fig. 3. The remaining face of each specimen was equipped with a transducer to record horizontal strains during testing for a different study. A custom adjustable frame was designed and built to quickly and securely hold all the transducers in position without inhibiting their measuring ability.

All tests were carried out with load control while measurements were recorded with a sampling rate of 5 Hz. A load cell with a maximum compression force of 200 kN was used for all tests except for those on lime mortar specimens. These were tested using a load cell with a maximum capacity of 10 kN. During the cyclic loading procedure, loads were applied with a constant velocity of 0.25 kN/s in between successive nominal levels. As shown in Fig. 3, clamp-on strain transducers with an effective minimum resolution of approximately 0.013 $\mu\text{m}/\text{mm}$ were used to measure strains during all tests.

2.2.3. Derivation of static elastic modulus from raw data

Once the experiments were completed, the compression loads recorded during each test were converted to stresses through division by the cross-sectional area of the specimen. Data points corresponding to the parts of the third loading cycle held at the nominal lower (σ_a) and upper (σ_b) stress levels were then extracted from all the stress and strain measurements. In order to ensure that there were no inconsistencies during loading and that the stresses were effectively held at the nominal levels, the coefficients of variation among extracted data points corresponding to each specific level were verified. Since they never exceeded 0.25%, the loading of all the specimens was deemed as being consistent with the scheme shown in Fig. 2 and the mean stress recorded during the period for which the loading was held at a nominal level was taken as the central tendency.

Similarly, each representative measured strain value corresponding to a nominal stress level was computed as the mean of the strains recorded during the period for which the stress was held at that level. However, since surface strain measurements of brittle materials are prone to error, two important verifications were carried out to ensure erroneous measurements were not included in the computation of the elastic modulus. First, measurements from a specific strain transducer for a particular test were discarded if the magnitude of the mean strain for a nominal stress level was lower than the minimum resolution of the transducer (0.013 $\mu\text{m}/\text{mm}$). Measurements smaller than this hardware limitation can actually be considered as being equivalent to not registering any strain. Second, measurements from a specific strain transducer for a particular test were also discarded if the coefficient of variation among extracted data points corresponding to the same nominal level exceeded 1%. This requirement was established to identify apparent sudden changes of strain that were recorded while the applied load was kept constant, such as the one shown in Fig. 4.

This cross-check of the experimental readings is introduced in this research as it proves to be of paramount importance in experimental tests evaluating the deformation characteristics of masonry components. Identifying and eliminating such erroneous values, mostly due to possible contact problems between the strain

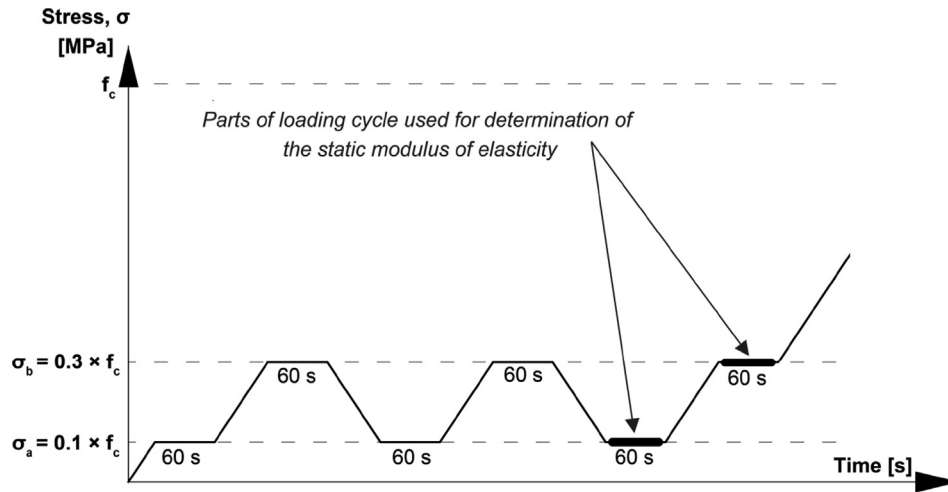


Fig. 2. Loading scheme employed for tests to determine the static elastic modulus of specimens.

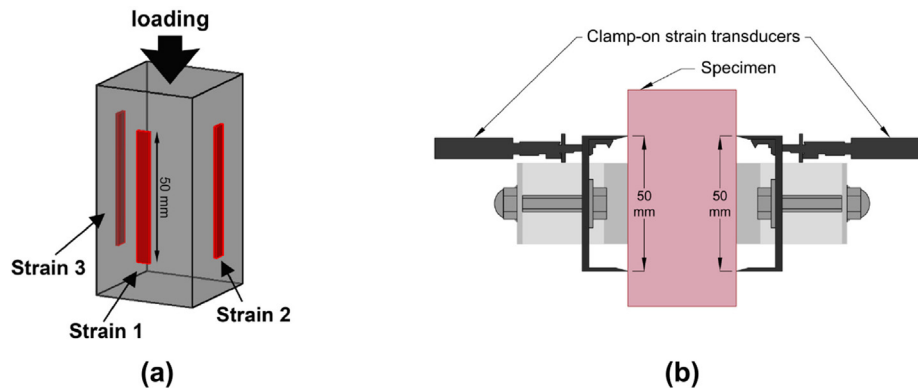


Fig. 3. (a) Schematic diagram showing locations at which strain was measured during testing of each specimen. (b) Cross-section across clamp-on strain transducers placed on opposing faces of a specimen.

transducer and the specimen's surface, can have a significant effect on the accuracy of the final estimates of the static elastic modulus. In this case, as a result of the filtering procedure described in the previous paragraph, out of the 168 specimens tested, at least one strain measurement was discarded from 22 specimens.

Once these erroneous strain values were eliminated, the strain difference between the nominal upper and lower stress levels

was computed for each strain transducer for every test. The mean of the three vertical strain changes measured across different faces of each specimen then needs to be computed before E_{st} can be estimated. Naturally, in case one or more of the strain measurements across a particular face had been deemed to be erroneous, they have to be excluded from the computation of the mean strain change. However, before proceeding with the estimation of E_{st} ,

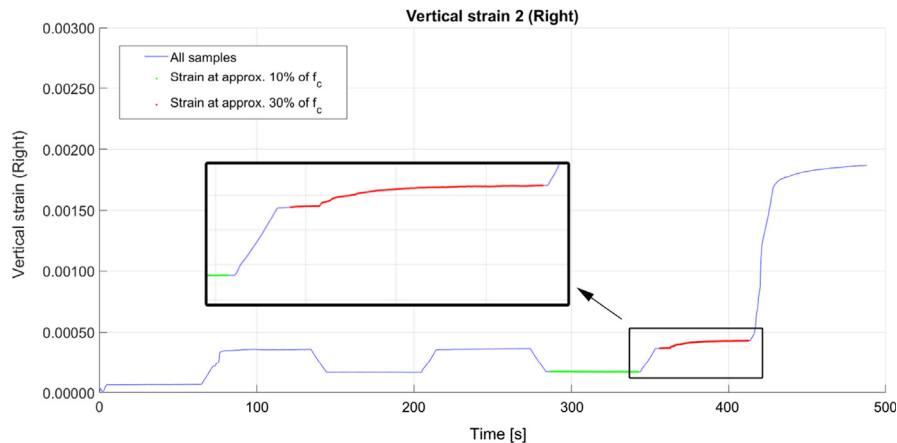


Fig. 4. Example of an unsteady strain measurement deemed as being erroneous.

for some of the specimens still left with three non-erroneous strain measurements, a clear outlier could be identified as shown in Fig. 5. These outliers most likely occurred due to the presence of very localised heterogeneities close to the surface across which the strain was being measured and are therefore not representative of the true material behaviour. Hence, although the treatment of outliers is rarely straightforward, in this particular case, it can be deemed that a better estimate of the actual E_{st} can be obtained by excluding such outliers from the data set.

Although there exists several statistical procedures to deal with outliers in data sets, many of them are not effective when only a small number of observations are available. As such, most common methods would not perform well for the case of the measured strain changes since only 3 observations are available. However, a well-known procedure, known as Dixon's Q test [43], has been developed specifically for such cases. It relies on the computation of a Q ratio for each observation as follows:

$$Q = \frac{x_n - x_{n-1}}{\omega} \quad (7)$$

Where x_n is the observation for which the Q ratio is being computed, x_{n-1} is the nearest neighbour of x_n and ω is the range of the set of observations defined as the difference between the largest and the smallest observation.

The conventional implementation of the Q test involves excluding observations with Q ratios greater than tabulated rejection thresholds corresponding to a specific confidence level assuming a normally distributed population. As can be seen from Eq. (7), the Q ratio is normalised by the range which is considered as one of the most efficient measures of dispersion for a small number of observations [43]. It is therefore based solely on the relative distances between observations irrespective of the magnitude of the range. As a result, the test tends to penalise outliers more strictly in a less dispersed sample. This is a disadvantage in the case of the measured strain changes since the desired outcome is to exclude an outlier that differs significantly from the other measurements in terms of magnitude rather than excluding one that differs relatively more than the others within a small range. To overcome this limitation, for each set of measured strain changes, the magnitude of the dispersion (measured by the range) was compared to the magnitude of the central tendency (measured by the mean). If the range was found to be greater than 75% of the mean value, the observation with the greatest Q ratio, computed as shown in Eq. (7), was excluded from the data set. Using this procedure, a single outlying value for the strain change was excluded for 10 of the 168 specimens tested as part of this research.

Subsequently, the static elastic modulus E_{st} of each tested specimen was computed using the following expression:

$$E_{st} = \frac{\overline{\sigma}_b - \overline{\sigma}_a}{\epsilon_{diff}^*} \quad (8)$$

Where $\overline{\sigma}_a$ and $\overline{\sigma}_b$ refer to the mean stresses recorded during the third loading cycle at the lower and upper nominal levels respectively while ϵ_{diff}^* refers to the mean strain change measured between the two nominal stress levels across 3 different specimen faces excluding any identified outliers or erroneous values.

2.3. Experimental determination of dynamic modulus

The complete procedure to obtain reliable estimates of the dynamic elastic properties of brick masonry constituents, as well as an extensive discussion on the factors that can influence them, has already been presented in a previous publication [11] and will not be reiterated in this article. However, a brief summary of the relevant processes employed to estimate the dynamic elastic modulus compared to static ones is provided in this section.

As described in [11], the most reliable estimate of the dynamic elastic modulus across a brick's length can be obtained through Impulse Excitation of Vibration (IEV) testing. This test method relies on identifying natural frequencies corresponding to specific resonant modes that are isolated through particular test set-ups. Basically, the tests consist of applying an impulse at a particular location on a specimen constrained by specific boundary conditions such as the ones shown in Fig. 6(a). At the same time, the resulting acceleration of the specimen is recorded using a lightweight accelerometer fixed along nodal lines to minimise mass loading effects. The resonant frequency corresponding to each isolated mode of vibration is then extracted from the acquired vibration signatures using Frequency Domain Decomposition (FDD). Analytical expressions exist that relate specific resonant frequencies to the elastic properties of the material and the mass and dimensions of the specimen under test. Hence, once the required resonant frequencies are identified, dynamic elastic properties can be estimated by making use of the analytical expressions. The dynamic elastic modulus across a brick's length is related to the flexural vibration mode which can be isolated and identified using the test set-up shown in Fig. 6(a). This method was also used to estimate the dynamic modulus of the mortar specimens compared to the static ones in this study.

In order to estimate the elastic modulus across a brick's width, Ultrasonic Pulse Velocity (UPV) testing with compression waves was used instead. This method relies on measuring the travel time

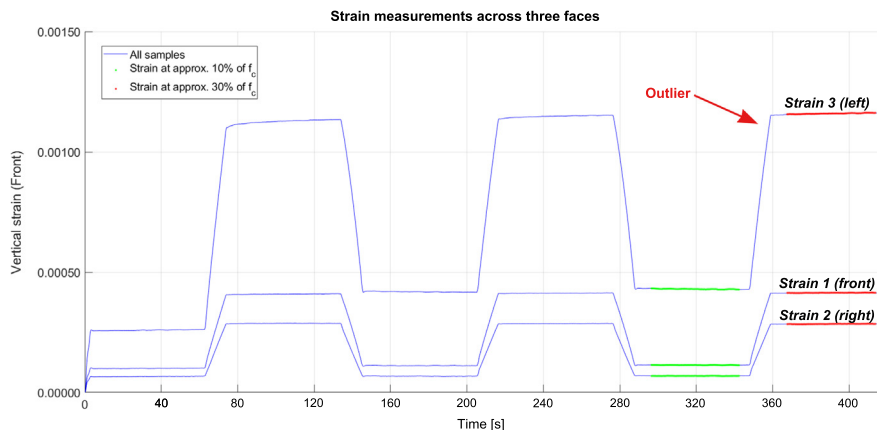


Fig. 5. Example of an outlier among 3 strain measurements across different faces of a specimen.

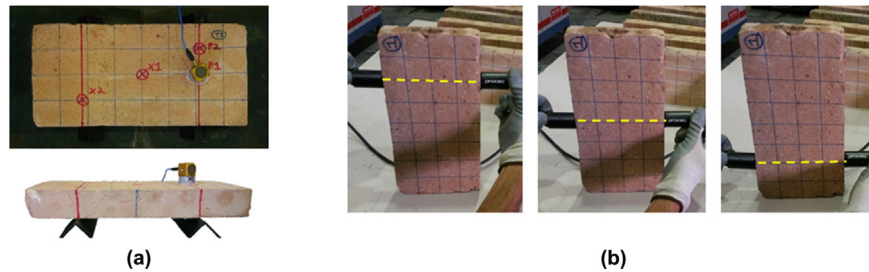


Fig. 6. (a) Flexural test set-up used for the estimation of the dynamic elastic modulus of mortars and across the length of bricks through Impulse Excitation of Vibration (IEV) testing. (b) Locations at which the travel time was measured for the estimation of the dynamic elastic modulus across the width of bricks using Ultrasonic Pulse Velocity (UPV) tests [11].

of an ultrasonic compression wave across a known length of the material. Because there exist analytical expressions relating the velocity of mechanical waves to elastic properties of the material, the measured travel time can be used to compute the velocity of the pulse and subsequently to estimate the elastic modulus across a brick's length. The procedure employed in [11] involves measuring the travel time and computing the corresponding pulse velocity at the three locations shown in Fig. 6(b). The average of these three velocities is then used for the estimation of the dynamic elastic modulus across the width of each brick. It is important to note that material isotropy is a fundamental assumption of the analytical expressions employed. As such, no estimates of the dynamic elastic modulus across the width could be obtained for bricks produced by extrusion (groups II and IV) since they presented a clearly anisotropic character [11].

3. Experimental results

For each brick, the average value of static elastic moduli of specimens extracted across the brick length was taken to represent the static elastic modulus of the brick across its length ($E_{st,L}$). Similarly, the average static elastic modulus of specimens extracted across a brick's width was taken as the static elastic modulus of the brick across its width ($E_{st,W}$). Each one of these values were compared to the most reliable corresponding dynamic elastic modulus (E_{dy}) of the same brick, which had previously been estimated using impulse excitation of vibration (IEV) or ultrasonic pulse velocity (UPV) methods [11]. As described in [11], estimates of the dynamic elastic modulus obtained using IEV are more reliable and representative of the elastic modulus of the entire specimen when compared to UPV estimates. Hence, values estimated using IEV were utilised for the comparison of elastic moduli across a brick's length and for mortar specimens. Nevertheless, it is worth mentioning that there is generally a good agreement between UPV and IEV estimates for isotropic constituent materials as described in [11]. Since the IEV methods utilised cannot provide estimates for the dynamic elastic modulus across a brick's width [11], UPV estimates were used for comparison to $E_{st,W}$.

For all specimen groups consisting of handmade bricks formed in moulds, except group V(b) (see Section 2.2.1), both $E_{st,L}$ and $E_{st,W}$ were computed. Due to their anisotropic nature, no dynamic elastic modulus across the width of bricks produced by extrusion was available for comparison (specimen groups II and IV) [11]. Therefore, $E_{st,W}$ of these bricks could not be used to better understand the relationship between E_{st} and E_{dy} . This resulted in a data set containing 35 pairs of E_{st} and E_{dy} estimates across brick lengths and 18 across brick widths. The average values of $E_{st,L}$ and $E_{st,W}$ for each specimen group are summarised in Tables 3 and 4 along with the average dynamic modulus values corresponding to the same specimen group and material direction.

In contrast, for mortars, a single average E_{st} is taken from all the tested prisms belonging to a specimen group. Thus only 3 new pairs of E_{st} and E_{dy} estimates are added to the data set containing 53 pairs of estimates from different brick specimens. The E_{st} values are shown together with the dynamic elastic modulus for the corresponding mortar type in Table 5.

As can be expected, the coefficient of variation (CV) of E_{st} is significantly greater than that of the corresponding E_{dy} for almost all specimen groups. Nonetheless, the CV between specimens for E_{dy} can be seen to be marginally greater than that of E_{st} in 3 cases, namely for MB mortars and for groups V(a) and V(b) when the modulus is evaluated across brick lengths. In these cases, the CV of E_{dy} tends to be only 1% or 2% higher than that of E_{st} . It should also be noted that many specimens from these particular groups were found to have a relatively high level of heterogeneity and much of the dispersion between estimated moduli values can be attributed to this [11]. In contrast, for the remaining specimen groups, the coefficients of variation among specimens for E_{st} are twice as large on average as those for E_{dy} . In fact, the greatest relative difference between these coefficients of variation occurs for one of the most homogeneous specimen groups (group IV). This is a clear indication of the increased difficulty and propensity for error of the static tests.

In terms of the magnitudes of the elastic moduli, with the exception of $E_{st,L}$ for specimen groups V(a) and V(b), the representative static elastic modulus of a specimen group is found to be always lower than its dynamic counterpart. This is in agreement with the trend generally observed in previous studies of this relationship for rocks and concrete, as already described in Section 1. In fact, for both group V(a) and group V(b), the relative difference between E_{st} and E_{dy} is smaller than the coefficient of variation associated to either value. This shows that the estimate of the difference between the two material properties is smaller than the dispersion related to the measurement of either one. As such, although exceptional observations for which $\frac{E_{st}}{E_{dy}}$ ratios greater than 1 have been reported for the case of rocks [6], in this case, it can be said that these occurrences cannot be deemed significant and do not reflect the true relationship between these two properties.

If individual specimens are analysed, the estimated static elastic modulus is greater than the dynamic one for 11 of the 56 pairs of E_{st} and E_{dy} available for comparison. However, it should be noted that 7 of the 11 specimens are from group V(a) or V(b). Moreover, it should be noted that, even for individual specimens, the ratio $\frac{E_{st}}{E_{dy}}$ is very close to 1 for most cases.

Based on the 42 pairs of E_{st} and E_{dy} values available for handmade bricks formed by moulding, simple linear regression with the intercept fixed at 0 reveals that 0.85 is a reasonable estimate of the $\frac{E_{st}}{E_{dy}}$ ratio for such bricks. The same procedure reveals that 0.87 is a good estimate of this ratio for bricks produced by extrusion based on the 11 data points available. These preliminary

Table 3Comparison of average static ($E_{st,L}$) and dynamic (E_{IEV}) elastic modulus measured across brick lengths for each specimen group.

Specimen group	Static		Dynamic*		$\frac{E_{st}}{E_{dy}}$
	$E_{st,L}$ [MPa]	coeff. of variation	E_{IEV} [MPa]	coeff. of variation	
I(a)	6,287	27%	7,882	15%	0.798
I(b)	7,920	22%	7,931	17%	0.999
II	16,081	7%	18,313	1%	0.878
III	5,996	22%	7,107	13%	0.844
IV	13,249	16%	15,505	1%	0.855
V(a)	5,572	7%	5,475	8%	1.018
V(b)	4,736	28%	4,068	30%	1.164

* Determined according to the procedure reported in [11].

Table 4Comparison of average static ($E_{st,W}$) and dynamic ($E_{UPV,W}$) elastic modulus measured across brick widths for each specimen group.

Specimen group	Static		Dynamic*		$\frac{E_{st}}{E_{dy}}$
	$E_{st,W}$	coeff. of variation	$E_{UPV,W}$ [MPa]	coeff. of variation	
I(a)	5,504	23%	8,241	19%	0.668
I(b)	7,563	21%	8,189	15%	0.924
II	–	–	–	–	–
III	6,051	18%	8,717	7%	0.694
IV	–	–	–	–	–
V(a)	5,389	23%	6,336	7%	0.851
V(b)	–	–	–	–	–

* Determined according to the procedure reported in [11].

Table 5Comparison of average static (E_{st}) and dynamic (E_{IEV}) modulus of mortar specimen groups.

Specimen group	Static		Dynamic*		$\frac{E_{st}}{E_{dy}}$
	E_{st} [MPa]	coeff. of variation	E_{IEV} [MPa]	coeff. of variation	
MB	2,696	8%	3,987	10%	0.676
MIIB	2,666	29%	4,269	6%	0.625
MC	27,167	8%	28,954	5%	0.938

* Determined according to the procedure reported in [11].

observations suggest that a suitable relationship can be found for both bricks produced by extrusion and for those handmade in moulds. As such, they were analysed as a single data set in an attempt to better understand the underlying relationship between static and dynamic elastic moduli for such materials. The 2 pairs of E_{st} and E_{dy} values available for lime mortar specimens suggest an average $\frac{E_{st}}{E_{dy}}$ ratio of 0.65 for this material. On the other hand, MC specimens tested as part of this research suggest a $\frac{E_{st}}{E_{dy}}$ ratio of 0.94, which is in good agreement with findings from previous studies on Portland cement-based mixes [15]. The two available data points for lime mortars and the single one of cement mortar certainly cannot be used for in-depth studies specifically on these respective material types. However, it was deemed beneficial to evaluate how well they agree with a general relationship for brittle constituent materials typically used in brick masonry constructions. As such, they were also included in the data set analysed in the following section.

4. Comparison and discussion

Previous studies [19,6,27,30] have already established that including material compressive strength, density and even porosity can lead to improved prediction capability of correlation models. However, given the variability of the sample set in terms of chemical composition and manufacturing process, it is unclear as to whether or not including them as explanatory variables can limit the range of practical applications significantly. Hence, in

order to evaluate this, a correlation study was carried out between different combinations of explanatory variables that can be included in the prediction model (see Tables 6 and 7). Such a study was also carried out previously in order to assess which combinations of material density and dynamic modulus to use for the prediction of the static elastic moduli of rocks [6]. It is achieved by computing the Pearson correlation coefficient ($R_{X,Y}$) between different combinations of dependent and explanatory variables, as shown in Eq. (9). This coefficient is a dimensionless measure of linear dependence that can vary between -1 and $+1$, with absolute values closer to unity indicating a better correlation. The sign of the coefficient indicates the type of correlation. A negative sign indicates that an increase of one parameter leads to a decrease of the other whereas a positive sign indicates that the increase of one leads to an increase of the other.

$$R_{X,Y} = \frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y} \quad (9)$$

Where $R_{X,Y}$ is the Pearson correlation coefficient between the variables X and Y , $cov(X,Y)$ is the covariance between them and σ_X and σ_Y are their respective standard deviations.

First, the same explanatory variable combinations explored in [6] for rocks were tested on the sample set of brick masonry constituents. As shown in Table 6, this involves various combinations of the material density (ρ) and the dynamic elastic modulus (E_{dy}).

Previous studies have shown that the compressive strength of the material can influence the relation between static and dynamic

elastic moduli for concrete ([1,17,18]) and rocks [30]. It was therefore deemed relevant to assess the suitability of including it as an explanatory variable for the case of brick masonry constituents. As such, as shown in Table 7, a correlation study was also carried out involving the same configurations as shown in Table 6 but substituting the material density with its compressive strength (f_c).

Since some researchers [30] have reported improved correlation for some rock types when increasing levels of complexity are added to the prediction expressions, the suitability of including both ρ and f_c was also carried out. This was achieved by substituting ρ with $\rho \cdot f_c$ in all the configurations tested in [6] before calculating the correlation coefficients. The outcome is summarised in Table 8.

As can be seen from Tables 6–8, of all the explored combinations, the strength of the linear correlation is strongest directly between E_{st} and E_{dy} . Although it is clear that there is still a relatively strong linear correlation between f_c and E_{st} or $\rho \cdot f_c$ and E_{st} , none of the explored combinations with E_{dy} contribute to strengthening its linear correlation with E_{st} . As previously mentioned, this is most likely due to the variability of the sample set which reflects the diversity of typical brick masonry constituents. Different mortars have distinct chemical compositions and can harden through different setting reactions. Bricks can differ greatly according to the manufacturing process and ingredients from which they are made. Given this variability, and recognising that the heterogeneity of materials affect the static and dynamic moduli in different ways, it cannot be expected that there exists a single relation between the two moduli based on physical behaviour. However, the high value of the correlation coefficient between E_{st} and E_{dy} is a promising indicator that there exists a linear empirical relationship between the two that could be useful for many practical applications.

As such, simple linear regression was carried out between measured values of E_{st} and E_{dy} to identify the unknown parameters of the relationship between the two. To prevent the proposed model from predicting zero or negative values of the static elastic modulus from positive measured values of the dynamic one, the intercept was fixed as 0. This left the slope as the only unknown parameter to be identified through the regression. As a result of this procedure, the following expression is proposed to estimate the static elastic modulus of brick masonry constituents from measurements of the dynamic one.

$$E_{st} = 0.87E_{dy} \tag{10}$$

Table 6
Correlation matrix for various combinations of static (E_{st}) and dynamic (E_{dy}) moduli with density (ρ) of brick masonry constituents.

Variables	E_{dy}	$\log_{10}E_{dy}$	ρ	$\rho \sqrt{E_{dy}}$	ρE_{dy}	$\sqrt{\rho E_{dy}}$	$\log_{10}\rho E_{dy}$	$\rho \log_{10}E_{dy}$	$\frac{E_{dy}}{\rho}$	$\sqrt{\frac{E_{dy}}{\rho}}$
E_{st}	0.97	0.89	0.10	0.94	0.96	0.95	0.89	0.70	0.94	0.92
$\log_{10}E_{st}$	0.92	0.92	−0.05	0.88	0.88	0.91	0.91	0.60	0.92	0.93

Table 7
Correlation matrix for various combinations of static (E_{st}) and dynamic (E_{dy}) moduli with compressive strength (f_c) of brick masonry constituents.

Variables	E_{dy}	$\log_{10}E_{dy}$	f_c	$f_c \sqrt{E_{dy}}$	$f_c E_{dy}$	$\sqrt{f_c E_{dy}}$	$\log_{10}f_c E_{dy}$	$f_c \log_{10}E_{dy}$	$\frac{E_{dy}}{f_c}$	$\sqrt{\frac{E_{dy}}{f_c}}$
E_{st}	0.97	0.89	0.84	0.90	0.93	0.93	0.87	0.86	−0.32	−0.32
$\log_{10}E_{st}$	0.92	0.92	0.85	0.87	0.87	0.91	0.92	0.85	−0.43	−0.42

Table 8
Correlation matrix for various combinations of static (E_{st}) and dynamic (E_{dy}) moduli with density (ρ) and compressive strength (f_c) of brick masonry constituents.

Variables	E_{dy}	$\log_{10}E_{dy}$	$\rho \cdot f_c$	$\rho \cdot f_c \sqrt{E_{dy}}$	$\rho \cdot f_c \cdot E_{dy}$	$\sqrt{\rho \cdot f_c \cdot E_{dy}}$	$\log_{10}(\rho \cdot f_c \cdot E_{dy})$	$(\rho \cdot f_c) \log_{10}E_{dy}$	$\frac{E_{dy}}{\rho f_c}$	$\sqrt{\frac{E_{dy}}{\rho f_c}}$
E_{st}	0.97	0.89	0.87	0.93	0.94	0.94	0.87	0.89	−0.33	−0.33
$\log_{10}E_{st}$	0.92	0.92	0.86	0.87	0.85	0.91	0.92	0.86	−0.43	−0.41

Where E_{st} and E_{dy} refer to the static and dynamic moduli measured in consistent units.

Despite the simplistic nature of the proposed expression, the measured data are in very good agreement with its predictions. In fact, although the sample of measured dynamic elastic moduli ranges more than 26 GPa (from 2.7 GPa to 29 GPa), the entire 95% prediction interval of the proposed relation spans only 4.6 GPa (see Fig. 7). In spite of this, it is worth noting that specifically for the weakened hydraulic lime mortar used as part of this study, using a $\frac{E_{st}}{E_{dy}}$ ratio of 0.65 is recommended instead of the general proposed relationship.

It is clear to see that the proposed relationship is extremely close to one of the simplest and oldest empirical expressions proposed for concrete [16]. This relation suggests that the static modulus is approximately equivalent to $0.83E_{dy}$. As such, although it is unlikely that a single relation between E_{st} and E_{dy} for brittle materials can be developed based on physical behaviour, it is undeniable that there are many similarities in the relation of these properties for such materials. Hence, several existing relationships proposed for concrete and rocks were tested on the data set containing 56 pairs of measured E_{st} and E_{dy} for various brick masonry constituents. Most of these are shown in Fig. 7.

Although several metrics were used to assess the accuracy of the models, the standard error of the estimate (σ_e) is probably the one that is most easily interpreted since it is expressed in the units of the measurements. This parameter indicates approximately how large prediction errors are for your data set and is computed as follows.

$$\sigma_e = \sqrt{\frac{\sum_{i=1}^n (Y_i - Y'_i)^2}{n - k - 1}} \tag{11}$$

Where Y_i refer to actual measurements of E_{st} while Y'_i refer to model predictions. With n equal to the number of data sample points and k equal to the number of explanatory variables used in the model, $n - k - 1$ represents the number of degrees of freedom available for the computation of the error metric.

The final standard prediction errors for the different models used to predict E_{st} from E_{dy} are shown in Fig. 8.

It can be seen that the proposed simple empirical expression results in the smallest error. The second most accurate model is clearly that proposed by Lydon and Balendran (1986) [16]. This is

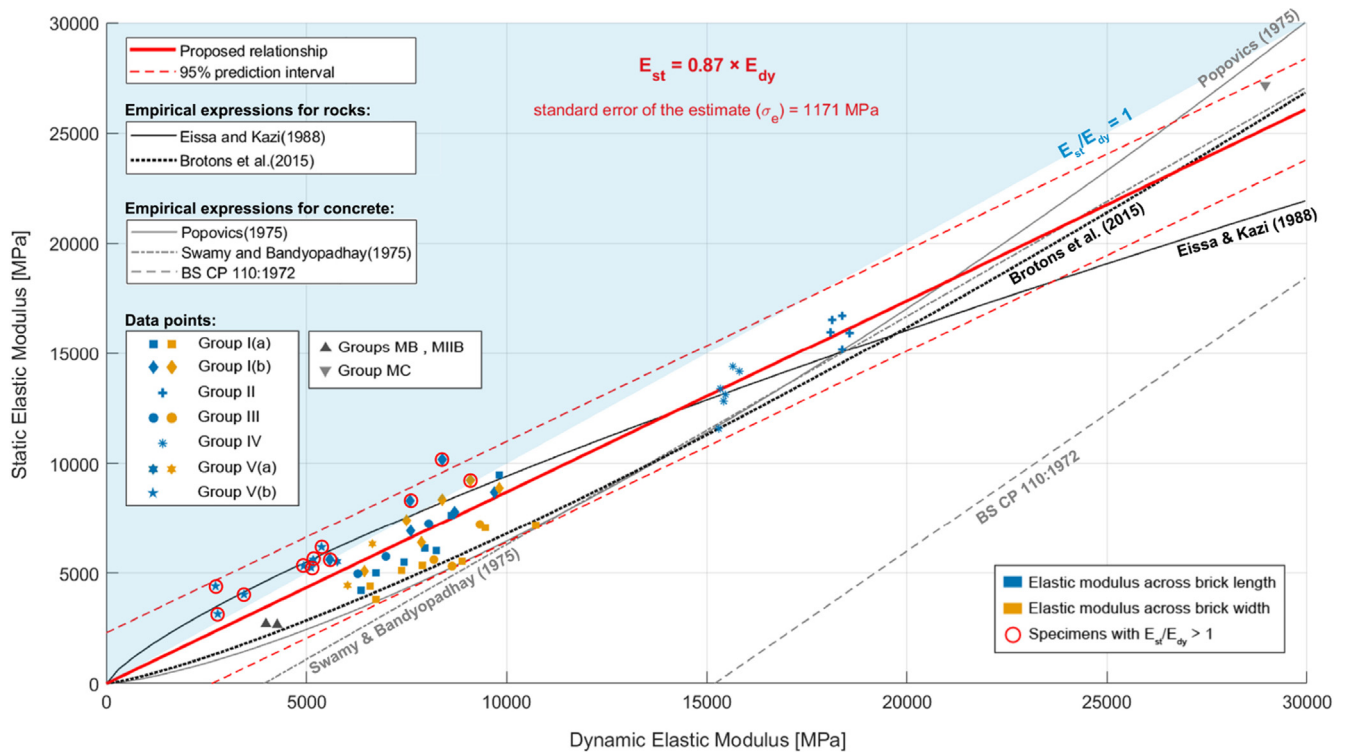


Fig. 7. Prediction of static elastic modulus (E_{st}) from dynamic elastic modulus (E_{dy}).

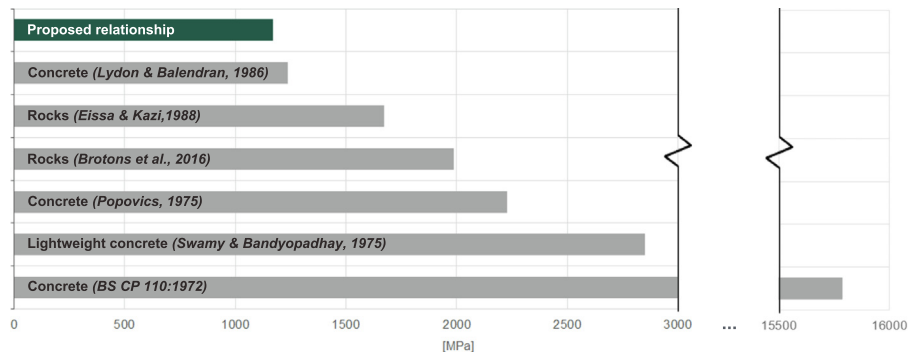


Fig. 8. Prediction errors of various existing expressions for estimating E_{st} from E_{dy} .

expected since it has the same form as the proposed single-parameter model with only a slight variation in the magnitude of the parameter. However, having been developed for concrete, the expression proposed by Lydon and Balendran was estimated from specimens with much higher stiffness. The closeness of the two identified models thus suggests that these simple expressions have a large range of applicability which can be very useful for practical applications. The least well-performing models appear to be linear ones developed for concrete that include a non-zero intercept. Although these work well for specific types of concrete, they are clearly not suitable for lower stiffness brittle materials such as the ones tested as part of this research. That being said, specifically for the cement mortar tested as part of this research, the expression proposed by Swamy and Bandyopadhyay (1975) for lightweight concrete [15] provides a better estimate of the measured static elastic modulus when compared to the proposed relationship. Although the nonlinear models that incorporate density are able to better represent the relation between E_{st} and E_{dy} for some small ranges of stiffness, it is clear that over the entire range of specimens tested, they are less accurate than the simpler proposed model. Finally, the nonlinear models developed for rocks appear to

be more applicable to the case of brick masonry constituents than the one developed for concrete. This is most likely due to the fact that the variability usually encountered in a sample of rocks more closely matches the variability of typical brick masonry constituents than that of concrete.

5. Conclusions

The research has proposed an empirical expression that can be used to estimate the static elastic modulus of typical brick masonry constituents from the dynamic modulus. Since the estimation of the latter parameter can be executed more quickly and reliably, the proposed expression can be used in a wide range of practical applications. To ensure that the expression would be meaningful for different masonry typologies, the tests have considered hydraulic lime and cement mortar, new bricks manufactured either by hand-made moulding or extrusion and bricks extracted from heritage buildings in Barcelona, Spain.

Several combinations of explanatory variables were investigated before selecting the final model. However, simply expressing the static elastic modulus as a ratio of the dynamic one appears to

be most effective over the range of materials tested. This most probably arises due to the large variability of brick masonry constituents in terms of isotropy, heterogeneity as well as chemical composition and manufacturing process. It is undeniable that incorporating other explanatory variables such as density or compressive strength would lead to more accurate models for specific types of brick masonry constituents. However, the proposed expression has proven to be applicable to a wide range of constituent materials with sufficient accuracy for many practical applications.

The most useful application of the proposed expression is that it provides a means to obtain very quick estimates of the static elastic modulus from whole bricks in a laboratory setting with minimal specimen preparation. This can save considerable time when compared to the tedious and lengthy cyclic static tests usually required to estimate this property. For bricks handmade in moulds, the study also revealed that the same expression can be applied to estimate the static elastic modulus across both the length or the width of bricks. Hence, although tests across other dimensions were not explicitly included in this study, it is deemed reasonable to assume that the proposed expression can be used together with appropriate dynamic tests to quickly estimate the static elastic modulus across the thickness of bricks formed by moulding. In addition, for the case of single-leaf brick masonry walls, the proposed relationship can be used together with ultrasonic pulse velocity (UPV) testing to quickly estimate the static modulus of elasticity of bricks in situ. In fact, the same procedure would even allow the estimation of the static elastic modulus of mortar in situ if some very thick joints can be identified. Since conventional UPV testing with compression waves is not valid for anisotropic material, such an in situ procedure cannot be directly extended to the case of walls built with bricks produced by extrusion.

The proposed empirical expression has been calibrated based on an accurate control over the measurements of the static modulus, as they usually exhibit higher scattering due to the relevant technical complexity of the tests. Although the applicability of the same suggested empirical expression cannot be ensured for materials differing significantly from those included in this research, the study has provided a general experimental methodology that may be replicated in the laboratory to derive alternative relationships.

CRedit authorship contribution statement

Nirvan Makoond: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **Albert Cabané:** Conceptualization, Methodology, Validation, Investigation, Data curation, Writing - review & editing. **Luca Pelà:** Conceptualization, Methodology, Validation, Investigation, Resources, Writing - review & editing, Supervision, Project administration, Funding acquisition. **Climent Molins:** Conceptualization, Methodology, Validation, Investigation, Resources, Writing - review & editing, Supervision, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] A.M. Neville, *Properties of concrete*, fifth ed., Pearson, 2011.
- [2] R.E. Phillee, Comparison of results of three methods for determining young's modulus of elasticity of concrete, *ACI J. Proc.* 51 (1) (1955) 461–470, <https://doi.org/10.14359/11690>.
- [3] K. Wesche, W. Manns, Résultats d'une enquête internationale sur la détermination du module d'élasticité du béton en compression, *Matériaux et Constructions* 3 (3) (1970) 179–196, <https://doi.org/10.1007/BF02478969>.
- [4] K.J. Bastgen, V. Hermann, Experience made in determining the static modulus of elasticity of concrete, *Matériaux et Constructions* 10 (6) (1977) 357–364, <https://doi.org/10.1007/BF02473733>.
- [5] E.I. Mashinsky, Differences between static and dynamic elastic moduli of rocks: Physical causes, *Russ. Geol. Geophys.* 44 (9) (2003) 953–959.
- [6] E. Eissa, A. Kazi, Relation between static and dynamic Young's moduli of rocks, *Int. J. Rock Mech. Mining Sci. Geomech. Abstracts* 25 (6) (1988) 479–482, [https://doi.org/10.1016/0148-9062\(88\)90987-4](https://doi.org/10.1016/0148-9062(88)90987-4).
- [7] L. Binda, C. Tiraboschi, S. Abbaneo, Experimental research to characterise masonry materials, *Masonry Int.* 10 (1997). URL: <https://www.masonry.org.uk/downloads/experimental-research-to-characterise-masonry-materials/>.
- [8] L. Binda, C. Tedeschi, P. Condoleo, Characterisation of Materials Sampled From Some My S'on Temples, in: 7th International Congress on Civil Engineering, 2006.
- [9] G. Baronio, L. Binda, C. Tedeschi, C. Tiraboschi, Characterisation of the materials used in the construction of the Noto Cathedral, *Constr. Build. Mater.* 17 (8) (2003) 557–571, <https://doi.org/10.1016/j.conbuildmat.2003.08.007>.
- [10] D.V. Oliveira, P.B. Lourenço, P. Roca, Cyclic behaviour of stone and brick masonry under uniaxial compressive loading, *Mater. Struct.* 39 (2) (2007) 247–257, <https://doi.org/10.1617/s11527-005-9050-3>.
- [11] N. Makoond, L. Pelà, C. Molins, Dynamic elastic properties of brick masonry constituents, *Constr. Build. Mater.* 199 (2019) 756–770, <https://doi.org/10.1016/j.conbuildmat.2018.12.071>.
- [12] L. Binda, M. Facchini, G. Mirabella Roberti, C. Tiraboschi, Electronic speckle interferometry for the deformation measurement in masonry testing, *Constr. Build. Mater.* 12 (5) (1998) 269–281, [https://doi.org/10.1016/S0950-0618\(98\)00009-9](https://doi.org/10.1016/S0950-0618(98)00009-9).
- [13] L. Pelà, E. Canella, A. Aprile, P. Roca, Compression test of masonry core samples extracted from existing brickwork, *Constr. Build. Mater.* 119 (2016) 230–240, <https://doi.org/10.1016/j.conbuildmat.2016.05.057>.
- [14] British Standards Institution (BSI), BS CP 110–1:1972; Code of practice for the structural use of concrete; Part 1: Design, materials and workmanship, 1972. <http://shop.bsigroup.com/en/ProductDetail/?pid=000000000010197621..>
- [15] R.N. Swamy, A.K. Bandyopadhyay, The elastic properties of structural lightweight concrete, *Proc. Inst. Civ. Eng.* 59 (3) (1975) 381–394, <https://doi.org/10.1680/jicep.1975.3671>.
- [16] F. Lydon, R. Balendran, Some observations on elastic properties of plain concrete, *Cem. Concr. Res.* 16 (3) (1986) 314–324, [https://doi.org/10.1016/0008-8846\(86\)90106-7](https://doi.org/10.1016/0008-8846(86)90106-7).
- [17] T. Takabayashi, Comparison of dynamic Young's modulus and static Young's modulus for concrete, RILEM international symposium on nondestructive testing of materials and structures 1 (Paris) (1954) 34–44.
- [18] B.J. Lee, S.-H. Kee, T. Oh, Y.-Y. Kim, Evaluating the dynamic elastic modulus of concrete using shear-wave velocity measurements, *Adv. Mater. Sci. Eng.* 2017 (2017) 1–13, <https://doi.org/10.1155/2017/1651753>.
- [19] S. Popovics, Verification of relationships between mechanical properties of concrete-like materials, *Matériaux et Constructions* 8 (3) (1975) 183–191, <https://doi.org/10.1007/BF02475168>.
- [20] M. King, Static and dynamic elastic properties of rocks from the Canadian shield, *Int. J. Rock Mech. Mining Sci. Geomech. Abstracts* 20 (5) (1983) 237–241, [https://doi.org/10.1016/0148-9062\(83\)90004-9](https://doi.org/10.1016/0148-9062(83)90004-9).
- [21] W. b Heerden, General relations between static and dynamic moduli of rocks, *Int. J. Rock Mech. Mining Sci. Geomech. Abstracts* 24 (6) (1987) 381–385, [https://doi.org/10.1016/0148-9062\(87\)92262-5](https://doi.org/10.1016/0148-9062(87)92262-5).
- [22] B. Christaras, F. Auger, E. Mosse, Determination of the moduli of elasticity of rocks. Comparison of the ultrasonic velocity and mechanical resonance frequency methods with direct static methods, *Mater. Struct.* 27 (4) (1994) 222–228, <https://doi.org/10.1007/BF02473036>.
- [23] L. L. Lacy, Dynamic Rock Mechanics Testing for Optimized Fracture Designs, in: SPE Annual Technical Conference and Exhibition, Society of Petroleum Engineers, 1997. doi:10.2118/38716-MS.

- [24] P. Horsrud, Estimating mechanical properties of shale from empirical correlations, *SPE Drill. Completion* 16 (02) (2001) 68–73, <https://doi.org/10.2118/56017-PA>.
- [25] M. Ciccotti, F. Mulargia, Differences between static and dynamic elastic moduli of a typical seismogenic rock, *Geophys. J. Int.* 157 (1) (2004) 474–477.
- [26] J. Martínez-Martínez, D. Benavente, M.A. García-del Cura, Comparison of the static and dynamic elastic modulus in carbonate rocks, *Bull. Eng. Geol. Environ.* 71 (2) (2012) 263–268, <https://doi.org/10.1007/s10064-011-0399-y>.
- [27] V. Brotons, R. Tomás, S. Ivorra, A. Grediaga, Relationship between static and dynamic elastic modulus of calcarenite heated at different temperatures: the San Julián's stone, *Bull. Eng. Geol. Environ.* 73 (3) (2014) 791–799, <https://doi.org/10.1007/s10064-014-0583-y>.
- [28] A.R. Najibi, M. Ghafoori, G.R. Lashkaripour, M.R. Asef, Empirical relations between strength and static and dynamic elastic properties of Asmari and Sarvak limestones, two main oil reservoirs in Iran, *J. Petrol. Sci. Eng.* 126 (2015) 78–82, <https://doi.org/10.1016/j.petrol.2014.12.010>.
- [29] W. Fei, B. Huiyuan, Y. Jun, Z. Yonghao, Correlation of dynamic and static elastic parameters of rock, *Electron. J. Geotech. Eng.* 21 (04) (2016) 1551–1560.
- [30] V. Brotons, R. Tomás, S. Ivorra, A. Grediaga, J. Martínez-Martínez, D. Benavente, M. Gómez-Heras, Improved correlation between the static and dynamic elastic modulus of different types of rocks, *Mater. Struct.* 49 (8) (2016) 3021–3037, <https://doi.org/10.1617/s11527-015-0702-7>.
- [31] Y. Z. Totoev, J. M. Nichols, A Comparative Experimental Study of the Modulus of Elasticity of Bricks and Masonry, in: 11th International Brick/Block Masonry Conference, no. October, 1997..
- [32] J.M. Nichols, Y.Z. Totoev, Experimental determination of the dynamic Modulus of Elasticity of masonry units, in: 15th Australian Conference on the Mechanics of Structures and Materials (ACMSM), 1997.
- [33] European Committee for Standardization (CEN), EN 772-1. Methods of test for masonry units. Part 1: Determination of compressive strength, 2010..
- [34] J. Segura, D. Aponte, L. Pelà, P. Roca, Influence of recycled limestone filler additions on the mechanical behaviour of commercial premixed hydraulic lime based mortars, *Constr. Build. Mater.* 238 (2020) 117722, <https://doi.org/10.1016/j.conbuildmat.2019.117722>.
- [35] C. Boulay, S. Staquet, M. Azenha, A. Deraemaeker, M. Crespini, J. Carette, J. Granja, B. Delsaute, C. Dumoulin, G. Karaïskos, Monitoring elastic properties of concrete since very early age by means of cyclic loadings, ultrasonic measurements, natural resonant frequency of composite beam (EMM-ARM) and with smart aggregates, in: VIII International Conference on Fracture Mechanics of Concrete and Concrete Structures, FraMCoS-8, no. March, 2013..
- [36] M. Ramesh, M. Azenha, P.B. Lourenço, Study of Early Age Stiffness Development in Lime-Cement Blended Mortars, in: RILEM Bookseries, vol. 18, Springer, Netherlands, 2019, pp. 397–404. doi:10.1007/978-3-319-99441-3_42..
- [37] European Committee for Standardization (CEN), EN 1015-11. Methods of test for mortar for masonry – Part 11: Determination of flexural and compressive strength of hardened mortar, 2019..
- [38] A. Földi, Effects influencing the compressive strength of a solid, fired clay brick, *Periodica Polytechnica Civil Eng.* 55 (2) (2011) 117, <https://doi.org/10.3311/pp.ci.2011-2.04>.
- [39] R.W. Zimmerman, The effect of microcracks on the elastic moduli of brittle materials, *J. Mater. Sci. Lett.* 4 (12) (1985) 1457–1460, <https://doi.org/10.1007/BF00721363>.
- [40] European Committee for Standardization (CEN), EN 12390-13. Testing hardened concrete – Part 13: Determination of secant modulus of elasticity in compression, 2013..
- [41] European Committee for Standardization (CEN), EN 14580. Natural stone test methods – Determination of static elastic modulus, 2005..
- [42] European Committee for Standardization (CEN), EN 1052-1. Methods of test for masonry – Part 1: Determination of compressive strength, 1998..
- [43] R.B. Dean, W.J. Dixon, Simplified statistics for small numbers of observations, *Anal. Chem.* 23 (4) (1951) 636–638, <https://doi.org/10.1021/ac60052a025>.